SIGNIFICANTLY-DIFFERENT WORD SQUARES

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In a May 1993 **Word Ways** article, Donald Knuth says that the reason his computer found so many 5x5x5 word cubes is that so many five-letter words can be "tweaked" into others. Each tweak or combination of tweaks can produce a new cube, only slightly different from the old one. All of us who have been finding word squares understand. The large numbers reported by Chris Long in the February 1993 **Word Ways** reflects this. To eliminate the effect of tweaking along the major diagonal (where a tweak always produces a new square), I devised the following procedure. I have been thinking about this for some time in regard to speeding the search for word squares; it can be applied to cubes equally well.

To start, my computer used 4839 common five-letter words to find 178,415 squares in 5.0 minutes. I then used a Carrollian ladder program to create a Boolean catalog. A number, BOO(x), was assigned to each word typically as follows. I illustrate with a subject word, CARES. BARES joins CARES in position 1; since B comes before C in the alphabet, mark BOO(CARES) in position 1. CORES joines CARES in position 2, but since O does not come before A, do not mark BOO(CARES). There are several words that mark BOO(CARES) in position 3. CARDS marks position 4, and CARED marks position 5. CARES ends up with a BOO of 10111 (decimal 23), CORES and CURES are similarly scored 31. BARED gets scored 4,being marked only by BAKED, which is scored 0.

In the second round, the computer ran all 4,839 words through a program which when looking for the nth word said if BOO(x) has a 1 in the nth position, do not use word x here. This program found 40,888 squares in only 2.3 minutes. Here are six squares sampled from forty (every thousandth square was printed out). Note that LEASE can be tweaked to LEAST in the first square, but the latter would not have been allowed. ASTER was allowed in the third square because neither ASKER nor ASPER were included in the basic list.

C A M E L	D R I P S	K V A S S	M U F T I	S O A P S	V I L L A
A G A V E	R A D I O	V I S I T	U H L A N	O N I O N	I D E A L
M A F I A	I D E A L	A S T E R	F L A S K	A I S L E	L E A N T
E V I L S	P I A N O	S I E G E	T A S T E	P O L K A	L A N C E
L E A S E	S O L O N	S T R E P	I N K E D	S N E A K	A L T E R

And then, for the sake of variety, I required the program to use five new words for each new square (in addition to the BOO test). I only got 225 squares this way; here are the last ones found.

SCUFF	SIGMA	SPRAT	SPURT	STRAW	YACHT
COPRA	INNER	PLAZA	PUREE	TRIPE	ARRAY
UPSET	GNATS	RADIX	URBAN	RIGID	CROOK
FREES	METRO	AZIDE	REACT	APING	HAOLE
FATSO	ARSON	TAXED	TENTH	WEDGE	TYKES

For readers who are not familiar with computer logic, the word Boolean refers to the fact that my programs use Boolean AND/OR logic to mark and test the words.

Having catalogued the five-letter words, I decided to see what happens with cubes. Before proceeding, I reduced my list to 4712 words by eliminating a few that didn't seem to be common enough. It was not difficult to expand a computer program for squares into one for cubes; the resultant worked fine. Using the BOO logic described above, it found 243 cubes in a little more than three hours. Without the BOO, it might have found a thousand or so, in a much longer time.

Before presenting a few findings, let me describe the architecture of a cube. A 5x5x5 cube contains fifteen different words. When we describe the cube in the following manner, ten of the fifteen words are printed twice (not used twice). Letters along the main diagonal are in boldface. These are the ones to which the BOO logic was applied.

WORD 1	WORD 2	WORD 3	WORD 4	WORD 5
WORD 2	W**O**RD 6	WORD 7	WORD 8	WORD 9
WORD 3	WORD 7	WO**R**D10	WORD11	WORD12
WORD 4	WORD 8	WORD11	WORD**D**13	WORD14
WORD 5	WORD 9	WORD12	WORD14	WORD**15**

Here are some cubes (asterisks indicate tweakable letters):

*	*	*	*	*
abbot	beach	baste	octet	theta
balsa	amahs	lapis	shite	asset
cable	ables	bloat	leave	ester
dodge	dboes	doses	geese	esses
ennui	nears	nasal	urate	isled
fiche	isles	cleat	heave	ester
grade	rubes	about	deuce	ester
heaps	eclat	alone	panda	stead
japan	alarm	pavid	amide	nodes
kaput	amass	pasha	usher	tsars
legit	evade	gales	ideal	tesla

eagle	aglow	clove	hewer	slope
moron	arena	honor	snare	pedal
bleat	lento	eaten	stone	onion
esses	bruin	outdo	eider	snort
eclat	alive	raven	steno	siren
sweat	lento	eaten	stone	endow
usurp	burro	error	spore	orbit
chive	liras	avail	tesla	orbit
lilac	alone	mania	ocean	vowel
mulct	altar	scale	strep	stave
vigas	agent	dance	ester	levee

toper	ewers	evens	terse	arses
inane	sales	tonic	erect	tests
atone	toned	venom	enemy	redye
solar	semis	erase	strep	
atone	towel	venom	enema	relax
urine	totes	conic	erect	rests
naive	ester	diver	alert	darts
inert	delta	dirge	eater	snare
haven	arena	elect	rents	spasm
steam	acnes	lease	armed	

These cubes use commoner words than those of Donald Knuth, and they don't contain his flaw of using a word twice. Based on simple probability arguments, we do not expect a word to be chosen twice but the nature of the system makes it happen often enough. The computer program used here contains specific logic to prevent duplications. This is usually not necessary for squares.